DERIVING RELATIONS FOR THE LEWIS NUMBER

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Experimentally confirmed theoretical relations are obtained for the ratio of the coefficients of heat and mass transfer (the Lewis number); an example of air and water in contact is given.

At present, expressions giving the Lewis number Le = $\alpha/(c_p\sigma)$ as dependent on other quantities describing heat and mass transfer during contact between a gas and liquid are not known.

A method for obtaining such relations is shown by using the following theoretical conditions.

The heat-balance equations given separately for the critical and final states of interacting media, for example, in the case of lowering the gas enthalpy, can be written as follows:

$$i_1 - i_{w1} = c_p(t_1 - t_{w1}) + r(d_1 - d_{w1}),$$
$$i_2 - i_{w2} = c_p(t_2 - t_{w2}) + r(d_2 - d_{w2}).$$

The heat due to overheating the liquid vapors is ignored as a negligible quantity compared with the enthalpy difference.

If the first equation is divided by the second and simple transformations are carried out, one obtains

$$\frac{i_1 - i_{w1}}{i_2 - i_{w2}} = \frac{t_1 - t_{w1}}{t_2 - t_{w2}} \frac{\xi_{w1}}{\xi_{w2}}, \tag{1}$$

where

$$\xi_{w1} = 1 + \frac{r}{c_p} \frac{d_1 - d_{w1}}{t_1 - t_{w1}}, \quad \xi_{w2} = 1 + \frac{r}{c_p} \frac{d_2 - d_{w2}}{t_2 - t_{w2}}.$$

Having taken the logarithms, relation (1) becomes

$$\ln \frac{i_1 - i_{w1}}{i_2 - i_{w2}} = \ln \frac{t_1 - t_{w1}}{t_2 - t_{w2}} + \ln \frac{\xi_{w1}}{\xi_{w2}}. \tag{2}$$

The individual terms of (2) are analyzed in more detail. Using the Merckel equation and the total heat balance (no external heat losses)

$$Gdi_g + Wc_w \frac{di_w}{m} = 0, \quad dQ = \sigma(i_g - i_w)\,dF \tag{3}$$

the following expressions are obtained:

$$i_g = i_w + \frac{Wc_w di_w}{m\sigma dF}, \quad i_w = i_g + \frac{Gdi_g}{\sigma dF}. \tag{4}$$

The temperature change of water in (3) was expressed by the proportionality coefficient $m = di_W/dt_W$ in terms of the corresponding enthalpy change of saturated air dt_W.

Relations (4) is differentiated with respect to dF,

$$\frac{di_g}{dF} = \frac{di_w}{dF} + \frac{Wc_w d^2 i_w}{m\sigma dF^2}, \quad \frac{di_w}{dF} = \frac{di_g}{dF} + \frac{Gd^2 i_g}{\sigma dF^2}, \tag{5}$$

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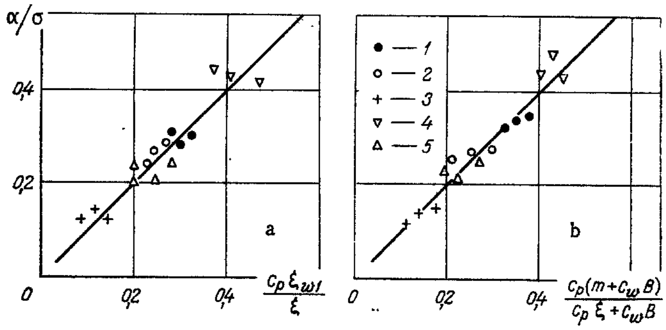

Fig. 1. Dependence of α/σ on $c_p\xi_{w1}/\xi$ (a) and $c_p(m + c_wB)/(c_p\xi + c_wB)$ (b): 1) t_1 = 12°, t_{m1} = 8°, t_{w1} = 3°; 2) 35, 28, and 11°; 3) 12, 8, and 25°; 4) 35, 28, and 26°; 5) 20, 12.6, and 20°.

and by substituting into the transformed equation (3),

$$\frac{Gdig}{\sigma dF} + \frac{Wc_w di_w}{m\sigma dF} = 0$$

the values of di_g/dF and di_w/dF from (5), one finally obtains

$$\frac{d^2ig}{dF^2} + A\frac{dig}{dF} = 0, \quad \frac{d^2i_w}{dF^2} + A\frac{di_w}{dF} = 0, \tag{6}$$

where $A = \sigma[(m/c_wW) + (1/G)]$.

The derivation of (4)-(6) was carried out under the usual assumption for this kind of problem – namely, that the coefficient of mass transfer σ is constant over the contact surface [1, 2].

The solution of (6) was obtained by a method similar to that used in the design of surface heat exchangers [3].

One of the possible results is

$$\ln\frac{i_1 - i_{w1}}{i_2 - i_{w2}} = \frac{\sigma F}{G}\left(1 + \frac{m}{c_wB}\right). \tag{7}$$

The method for obtaining an expression for the first term on the right of Eq. (2) which agrees with the above is

$$\ln\frac{t_1 - t_{w1}}{t_2 - t_{w2}} = \frac{\alpha F}{Gc_p}\left(1 + \frac{c_p\xi}{c_wB}\right), \tag{8}$$

where $\xi = (i_1 - i_2)/c_p(t_1 - t_2)$ is the coefficient of moisture loss obtained from the current values of the air parameters.

Equations (7) and (8) are now inserted into (2):

$$\frac{\sigma F}{G}\left(1 + \frac{m}{c_wB}\right) = \frac{\alpha F}{Gc_p}\left(1 + \frac{c_p\xi}{c_wB}\right) + \ln\frac{\xi_{w1}}{\xi_{w2}}. \tag{9}$$

From (9) the ratio of the coefficients of heat and mass transfer is found:

$$\frac{\alpha}{c_p\sigma} = \frac{m + c_wB}{c_p\xi + c_wB} - \frac{\ln(\xi_{w1}/\xi_{w2})}{\frac{\sigma F}{G}\left(1 + \frac{c_p\xi}{c_wB}\right)}. \tag{10}$$

Equation (10) can be rewritten as

$$\frac{\alpha}{c_p\sigma} = \frac{(m + c_wB)\ln\frac{t_1 - t_{w1}}{t_2 - t_{w2}}}{(c_p\xi + c_wB)\ln\frac{i_1 - i_{w1}}{i_2 - i_{w2}}}. \tag{11}$$

The same result can also be obtained in another way – by transforming the heat balance expressed with the aid of average logarithmic differences of enthalpy and temperature potentials of the media and by means of the coefficients α and σ:

$$\xi\alpha\Delta t_{\mathrm{m.l.}}F = \sigma\Delta i_{\mathrm{m.l.}}F.$$

Equation (11) can be written in the form

$$\frac{\alpha}{c_p\sigma} = \frac{m + c_w B}{c_p\xi + c_w B}\left(1 + \frac{\ln\dfrac{\xi_{w2}}{\xi_{w1}}}{\ln\dfrac{i_1 - i_{w1}}{i_2 - i_{w2}}}\right). \tag{12}$$

The numerical value of the logarithmic term in (12) for polytropic processes of the change in the state of the air approaches zero and can be neglected, since $\xi_{W1} \approx \xi_{W2}$; the latter follows from the analysis of the operation of reheating chambers for air conditioning setups [1, 4].

Consequently,

$$\frac{\alpha}{c_p\sigma} = \frac{m + c_w B}{c_p\xi + c_w B}. \tag{13}$$

For isoenthalpic processes the original expression (1) simplifies to

$$\frac{t_1 - t_{w1}}{t_2 - t_{w2}} = \frac{d_1 - d_{w1}}{d_2 - d_{w2}},$$

which implies that $\alpha/c_p\sigma_1 = 1$, where σ_1 is the heat-transfer coefficient due to the difference in the moisture content, which, in turn, is due to the dry and saturated air at water temperature.

A similar results can also be obtained from (13) (for $i_1 = i_2$, $m = 0$, and $\xi = 0$):

$$\alpha/c_p\sigma = \alpha/c_p\sigma_1 = 1.$$

It is found by transforming (13) that

$$\frac{\alpha}{c_p\sigma} = \frac{\xi_{w1}\left(1 - \dfrac{i_2 - i_{w2}}{i_1 - i_{w1}}\right)}{\xi\left(1 - \dfrac{t_2 - t_{w2}}{t_1 - t_{w1}}\right)}.$$

For $\xi_{W1} = \xi_{W2}$ it becomes

$$\alpha/c_p\sigma = \xi_{w1}/\xi. \tag{14}$$

The relations (13) and (14) are satisfied in processes in which the change of temperature fields and partial pressures is similar. In actual processes there is no equality between the corresponding moving forces and the boundaries of the flows, since in the case of nonstationary motion of the liquid drops they are realized on a discrete surface of inhomogeneous and multidisperse structure.

Nevertheless, qualitatively they correctly reflect the character of the changes of the ratio of the coefficients of heat and mass transfer. Thus, it follows from (14) that for $t_1 = t_{W1}$ [the parametric test being $T = (t_{M1} - t_{W1})/(t_1 - t_{M1}) = -1$) ξ_{W1} assumes an infinite value; if $\xi_{W1} = \xi_{W2}$, then ξ is also infinite. To be able to eliminate the ratio of two infinitely great quantities, one uses experimental data according to which for $T = -1$ one has $\alpha/\sigma = 1$, a negative value of ξ corresponding to the change of direction of the curve from a growing ratio α/σ to a decreasing one [2].

It should be noted here that (13) and (14) imply that

$$\xi_{w1} \neq m + c_w B, \quad \xi \neq c_p\xi + c_w B.$$

An experimental checkup on the relations (13) and (14) was carried out on a setup whose diagram was shown in [5]. Air and water were used as media. It can be seen from the obtained results shown in the Fig. 1a, b that the experimental points are satisfactorily concentrated close to the straight line (the scatter is ±30%). The greatest deviation is observed in the isothermic process of changing the air state or close to it.

It is our view that the above analysis can be used in the study of contact heat exchangers and in developing new methods for their design.

NOTATION

i is the enthalpy;
t is the temperature;
c_p, c_w are the specific heats of gas and liquid, respectively;
r is the specific heat of liquid evaporation;
α is the heat-transfer coefficient;
σ is the mass-transfer coefficient;
Q is the flux;
F is the contact surface;
G is the mass flow rate of liquid.

Indices

1 is the initial state;
2 is the final state;
w is the liquid;
g is the gas;
m.l. is the average logarithmic value;
Δ is the parameter difference;
M is the wet-bulb temperature.

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